

Write and **prove** a formula for $\sinh(x - y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.

SCORE: ____ / 6 PTS

$$\begin{aligned} & \sinh x \cosh y - \cosh x \sinh y \quad (2) \\ = & \left[\frac{e^x - e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x + e^{-x}}{2} \frac{e^y - e^{-y}}{2} \right] \quad (1) \\ = & \left[\frac{e^{x+y} + e^{x-y} - e^{y-x} - e^{-x-y} - (e^{x+y} - e^{x-y} + e^{y-x} - e^{-x-y})}{4} \right] \quad (2) \\ = & \frac{2e^{x-y} - 2e^{y-x}}{4} \\ = & \left[\frac{e^{x-y} - e^{-(x-y)}}{2} \right] \quad (1) \\ = & \sinh(x-y) \end{aligned}$$

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry.

SCORE: _____ / 4 PTS

- [a] Write that identity involving $\sinh x$ and $\cosh x$. **NOTE: You do NOT need to prove the identity.**

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

- [b] Divide both sides of that identity by $\cosh^2 x$ and simplify.

$$1 - \tanh^2 x = \operatorname{sech}^2 x \quad (1)$$

- [c] If $\sinh x = -7$, find $\coth x$.

$$\begin{aligned} \cosh^2 x - (-7)^2 &= 1 \\ \cosh^2 x &= 50 \end{aligned} \quad (1) \quad \text{EITHER IS OK}$$

$$\cosh x = \pm 5\sqrt{2} \quad (1/2)$$

SINCE $\cosh x > 0$, $(1/2)$

THEREFORE $\cosh x = 5\sqrt{2}$ $(1/2)$

$$\begin{aligned} \coth x &= \frac{\cosh x}{\sinh x} \\ &= \frac{-5\sqrt{2}}{7} \quad (1/2) \end{aligned}$$

Solve $\sinh x = 1$ by using the exponential definition of $\sinh x$ and an algebraic substitution $z = e^x$.

SCORE: ____ / 6 PTS

$$\frac{e^x - e^{-x}}{2} = 1$$

$$\left[\frac{z - \frac{1}{z}}{2} = 1 \right] \textcircled{1}$$

$$z - \frac{1}{z} = 2$$

$$z^2 - 1 = 2z$$

$$\left[z^2 - 2z - 1 = 0 \right] \textcircled{1}$$

$$z = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} \textcircled{\frac{1}{2}}$$

EITHER

IS OK

$$= \left[\frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \right] \textcircled{1}$$

$$z = e^x > 0 \textcircled{1}$$

$$\text{so } z = 1 + \sqrt{2}$$

$$e^x = 1 + \sqrt{2}$$

$$\left[x = \ln(1 + \sqrt{2}) \right] \textcircled{\frac{1}{2}}$$

Rewrite $\operatorname{csch}(3 \ln 2)$ in terms of exponential functions and simplify.

SCORE: _____ / 3 PTS

$$\left[\frac{2}{e^{3\ln 2} - e^{-3\ln 2}} \right] = \frac{2}{e^{\ln 8} - e^{\ln \frac{1}{8}}} = \left[\frac{2}{8 - \frac{1}{8}} \right] = \left[\frac{16}{63} \right]$$

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $g(f(x))$.

SCORE: ____ / 5 PTS

$$\begin{aligned} & \ln(\cosh x + \sqrt{\cosh^2 x - 1}) \quad (1) \\ &= \ln(\cosh x + \sqrt{\sinh^2 x}) \\ &= \ln(\cosh x + \sinh x) \quad (1) \\ &= \ln\left(\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}\right) \quad (1) \\ &= \ln \frac{2e^x}{2} \\ &= \ln e^x \quad (1) \\ &= x \quad (1) \end{aligned}$$